LIMITS
COURSE

LESSON 1
Introduction to limits of sequences.
Factoring out the highest power.

HOMEWORK

Part 1: TEST

Select the correct answer (only one is true).

Question 1

What can limits be taken of?

a) Functions and sequences
b) Finite sequences
c) Sequences and numbers
d) Sequences and number lines

Question 2

$a_n = 1 - n^2$

What is the fourth term of this sequence?

a) 4
b) n=4
c) -15
d) 17

Question 3

$|a_n - g|$

What does the above expression represent in the definition of the limit of a sequence?

a) The distance of a sequence term from the limit
b) An arbitrarily small ε
c) Approaching the limit
d) The distance of the index from the limit

Question 4

If the limit of a sequence turns out to be ∞, this means that…

a) the limit of the sequence does not exist
b) the sequence has an improper limit ∞
c) the sequence is arithmetic and increasing
d) the sequence may be decreasing

Question 5

$$\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n^2}\right)$$

What is the limit of the above sequence?

a) ∞
b) 0
c) 1
d) 2

Question 6

What do indeterminate symbols mean in sequence limits?

a) That before transformation the sequence has no limit
b) That we do not know what the sequence tends to and must transform it somehow
c) That before transformation the sequence diverges to ∞
d) That the sequence converges to 0 or diverges to ∞

Question 7

$$\lim_{n\to\infty}\left(\frac{3n^2+2n-1}{4n^2-n+5}\right)$$

How will the above sequence look after factoring out the highest powers?

a) $\lim_{n\to\infty}\left(\frac{n^2\left(3+\frac{2n}{n^2}-\frac{1}{n^2}\right)}{n^2\left(4-\frac{n}{n^2}+\frac{5}{n^2}\right)}\right)$

b) $\lim_{n\to\infty}\left(\frac{3n^2\left(\frac{2n}{3n^2}-\frac{1}{3n^2}\right)}{4n^2\left(-\frac{n}{4n^2}+\frac{5}{4n^2}\right)}\right)$

c) $\lim_{n\to\infty}\left(\frac{n\left(3n+\frac{2n}{n}-\frac{1}{n}\right)}{n\left(4n-\frac{n}{n}+\frac{5}{n}\right)}\right)$

d) $\lim_{n\to\infty}\left(\frac{n^2\left(3n+\frac{2n}{n^2}-\frac{1}{n^2}\right)}{n^2\left(4n-\frac{n}{n^2}+\frac{5}{n^2}\right)}\right)$

Question 8

$$\lim_{n\to\infty}\left(\frac{1-n^2}{1+n^2}\right)$$

How will the numerator of the above expression look after factoring out the highest power?

a) $n^2\left(\frac{1}{n^2}-1\right)$

b) $n^2\left(\frac{1}{n^2}+1\right)$

c) $n^2\left(1-\frac{1}{n^2}\right)$

d) $n^2\left(1+\frac{1}{n^2}\right)$

Question 9

$$\lim_{n\to\infty}\left(\frac{\sqrt[3]{n^3+5}+\sqrt{n^2-4}}{n+11}\right)$$

How should the numerator be transformed at this step of the problem?

a) Factor out from the entire numerator the highest power, i.e. n^3
b) Factor out n from the entire numerator
c) Factor out the highest powers that are under the roots
d) Divide both numerator and denominator by n^3

Question 10

$$(n-1)!$$

How can the above factorial be expanded?

a) $(n-1)!=(n-1)(n-2)(n-3)\ldots\cdot 2\cdot 1$
b) $(n-1)!=n(n-1)(n-2)(n-3)\ldots\cdot 2\cdot 1$
c) $(n-1)!=(n+1)n(n-1)(n-2)(n-3)\ldots\cdot 2\cdot 1$
d) $(n-1)!=\dfrac{n(n-1)(n-2)(n-3)\ldots\cdot 2\cdot 1}{n-1}$

Part 2: EXERCISES

Ex. 1

Solve the following limits:

1) $\lim_{n\to\infty}\left(2-\frac{4}{n}\right)$

2) $\lim_{n\to\infty}\left(\frac{-1}{n^2+5}\right)$

3) $\lim_{n\to\infty}\left(\frac{5}{n+\sqrt{n}}-1\right)$

4) $\lim_{n\to\infty}(2n+13)$

5) $\lim_{n\to\infty}\left(\frac{4n-11}{4n+5}\right)$

6) $\lim_{n\to\infty}\left(\frac{2n^2+5}{n^2}\right)$

7) $\lim_{n\to\infty}\left(\frac{-n^2+3n-1}{2n^2+5n+5}\right)$

8) $\lim_{n\to\infty}\left(\frac{n^3}{4n^3+4n^2+8n+16}\right)$

9) $\lim_{n\to\infty}\left(\frac{3-n^4}{n^4+1}\right)$

10) $\lim_{n\to\infty}\left(\frac{n+2}{n^2-n}\right)$

11) $\lim_{n\to\infty}\sqrt{\frac{13n+1}{52n}}$

12) $\lim_{n\to\infty}\left(\frac{2n^2-2+n}{1+2n-n^2}\right)^4$

13) $\lim_{n\to\infty}\left(\frac{n^3+n^2-2n+5}{5n^2+4n+3}\right)$

14) $\lim_{n\to\infty}\left(\frac{-7n^5-6n^4+122n^3-15n^2+7n-2}{21n^5+12n^3-501}\right)$

15) $\lim_{n\to\infty}\frac{(3n-1)(2n+2)}{n^2}$

16) $\lim\limits_{n\to\infty}\dfrac{(4n-1)^2}{3n+7}$

17) $\lim\limits_{n\to\infty}\sqrt{\dfrac{(3n-1)(3n+2)}{n^2-1}}$

18) $\lim\limits_{n\to\infty}\dfrac{3n-3}{\sqrt{n^2+1}}$

19) $\lim\limits_{n\to\infty}\dfrac{\sqrt{2n^2-1}+\sqrt{n^2+3n+3}}{\sqrt[3]{2n^3-1}}$

20) $\lim\limits_{n\to\infty}\dfrac{3^n+4^n}{4^n}$

21) $\lim\limits_{n\to\infty}\dfrac{5^n-2^n+10^n}{11^n+5^n}$

22) $\lim\limits_{n\to\infty}\dfrac{2^{3n+2}+6^{n-2}+3}{8^{n+2}+4^{n-1}+2^{2n+3}}$

23) $\lim\limits_{n\to\infty}\dfrac{2^{4n}-3^{2n+1}}{10^{n-1}+1}$

24) $\lim\limits_{n\to\infty}\dfrac{n!+(n+1)!}{n!+(n+2)!}$

25) $\lim\limits_{n\to\infty}\dfrac{\binom{n}{2}}{n^2+3n-1}$

END