LIMITS
COURSE

Lesson 2
Multiplying by the conjugate

HOMEWORK

Part 1: TEST

Select the correct answer (only one is true).

Question 1

The conjugate-multiplication method is similar to an operation taught in primary and secondary school. It is…

a) determining the monotonicity of a sequence
b) removing irrationality from the denominator
c) factoring out the highest power under a root sign
d) removing irrationality from the numerator

Question 2

$$\lim_{n\to\infty}\left(\sqrt{n}+\sqrt{n+1}\right)$$

Can we use the conjugate-multiplication method to compute the above limit?

a) No
b) Yes

Question 3

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n-1}\right)$$

At this stage of the problem, by what should the expression be multiplied?

a) $\dfrac{\sqrt{n-1}+\sqrt{n-1}}{\sqrt{n-1}+\sqrt{n-1}}$
b) $\dfrac{\sqrt{n+1}+\sqrt{n+1}}{\sqrt{n+1}+\sqrt{n+1}}$
c) $\dfrac{\sqrt{n-1}-\sqrt{n-1}}{\sqrt{n-1}-\sqrt{n-1}}$
d) $\dfrac{\sqrt{n+1}+\sqrt{n-1}}{\sqrt{n+1}+\sqrt{n-1}}$

Question 4

Which special-product (short product) formula do we use in the conjugate-multiplication method?

a) $a^2-b^2=(a-b)(a+b)$

b) $(a-b)^2=a^2-2ab+b^2$

c) $(a+b)^2=a^2+2ab+b^2$

Question 5

$$\lim_{n\to\infty}\left(\frac{4n}{\sqrt{3n^2+n+1}+\sqrt{3n}}\right)$$

After multiplying by the conjugate and tidying up, we obtained the above limit. What should be done at this stage of the problem?

a) Write the result, i.e. 0.
b) Cancel the common factor n in numerator and denominator.
c) Factor out the highest powers in the denominator (first under the root sign).
d) Write the result, i.e. 4.

Question 6

$$\lim_{n\to\infty}\left(\frac{\sqrt{n}-\sqrt{n-1}}{n^2-n}\right)$$

At this stage of the problem, by what should the above expression be multiplied?

a) $\dfrac{\sqrt{n}+\sqrt{n-1}}{\sqrt{n}+\sqrt{n-1}}$

b) $\dfrac{\sqrt{n}+\sqrt{n-1}}{n^2+n}$

c) $\dfrac{\sqrt{n}+\sqrt{n-1}}{\sqrt{n}+\sqrt{n-1}}\dfrac{n^2+n}{n^2+n}$

d) $\dfrac{\sqrt{n}+\sqrt{n+1}}{n^2+n}$

Question 7

$$\lim_{n\to\infty}\frac{\left(\sqrt{n^4+n^2}-n^2\right)\left(\sqrt{n^4+n^2}+n^2\right)}{\sqrt{n^4+n^2}+n^2}$$

What expression will appear in the numerator after applying the special-product formula?

a) $n^4+n^2+n^4$
b) $n^4+n^2-n^4$
c) $n^4+n^2-n^2$
d) $n^4-n^2-n^4$

Question 8

$$\lim_{n\to\infty}\frac{3}{\left(\sqrt{n}-\sqrt{n+7}\right)\left(\sqrt{n}+\sqrt{n+7}\right)}$$

What will the denominator of the above expression look like after applying the special-product formula?

a) -7
b) 7
c) $n+7$
d) $n-7$

Question 9

$$\lim_{n\to\infty}\frac{3n}{n\sqrt{1-\frac{2}{n}}+n\sqrt{1+\frac{4}{n}}}$$

How will this limit look after factoring the highest power out of the denominator?

a) $\lim_{n\to\infty}\frac{3n}{n\left(\sqrt{\frac{1}{n}-\frac{2}{n^2}}+\sqrt{\frac{1}{n}+\frac{4}{n^2}}\right)}$

b) $\lim_{n\to\infty}\frac{3n}{n\left(\sqrt{\frac{1}{n}-\frac{2}{n^2}}+\sqrt{1+\frac{4}{n}}\right)}$

c) $\lim_{n\to\infty}\frac{3n}{n\left(\sqrt{1-\frac{2}{n}}+\sqrt{1+\frac{4}{n}}\right)}$

d) $\lim_{n\to\infty}\frac{3}{n\left(\sqrt{1-\frac{2}{n}}+\sqrt{1+\frac{4}{n}}\right)}$

Question 10

When, in the conjugate-multiplication method, is it **not** necessary to factor out the highest powers?

a) When, after multiplying by the conjugate, we still get a limit with an indeterminate form.
b) When the radicals contain n only to the first power.
c) When, after multiplying by the conjugate, we obtain a finite numerical limit.
d) When, after multiplying by the conjugate, we no longer have an indeterminate form.

Part 2: EXERCISES

Ex. 1

Solve the following limits:

1) $\lim_{n\to\infty}\left(\sqrt{n+3}-\sqrt{n}\right)$
2) $\lim_{n\to\infty}\left(\frac{7}{\sqrt{n}-\sqrt{n+1}}\right)$
3) $\lim_{n\to\infty}\frac{\sqrt{n^2+10}-n}{n}$
4) $\lim_{n\to\infty}\left(2n-\sqrt{4n^2+4}\right)$
5) $\lim_{n\to\infty}\left(\sqrt{n^2+n}-n\right)$
6) $\lim_{n\to\infty}\left(\frac{3n}{n-\sqrt{n^2-n}}\right)$
7) $\lim_{n\to\infty}\frac{\sqrt{n^2+2n}-\sqrt{n^2-2n}}{5}$
8) $\lim_{n\to\infty}\left(\sqrt{2n^2-4n+7}-\sqrt{2}n\right)$
9) $\lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n}\right)$
10) $\lim_{n\to\infty}\frac{\sqrt{n^2+6}-n}{\sqrt{n^2+2}-n}$
11) $\lim_{n\to\infty}\left(\sqrt{n^4+n^2}-\sqrt{n^4-n^2}\right)$

END