LIMITS
COURSE

LESSON 3
Limits using the definition of *e*

HOMEWORK

Part 1: TEST

Select the correct answer (only one is true).

Question 1

The closest common approximation to the number *e* is…

a) 3,14
b) 3
c) 2,5
d) 2,7

Question 2

For which indeterminate form do we use the formula involving the constant *e*?

a) $\left[\frac{\infty}{\infty}\right]$
b) $\left[1^{\infty}\right]$
c) $\left[\infty-\infty\right]$
d) $\left[0^{\infty}\right]$

Question 3

How can the number e be defined?

a) As a certain sequence
b) As the limit of a certain sequence
c) As a particular rational number
d) As the difference of certain sequences

Question 4

$$a_n = \left(1 + \frac{1}{n}\right)^n$$

What is the limit of the above sequence?

a) e
b) 1
c) ∞
d) e^{∞}

Question 5

$$\lim_{n\to\infty}\left(1 + \frac{\boxed{?}}{n^2 + 3n}\right)^{n^2+3n} = e^{-4}$$

Which number should go in the blank box to make the equation true?

a) 4
b) -4
c) 3
d) -3

Question 6

$$\lim_{n\to\infty}\left(1+\frac{-5}{n-1}\right)^{n+5}$$

How can we transform the above limit so that we can apply the formula for the number *e*?

a) $\lim_{n\to\infty}\left(1+\frac{-5}{n-1}\right)^{n+5}=\lim_{n\to\infty}\left[\left(1+\frac{-5}{n-1}\right)^{n-1}\right]^{\frac{n-1}{n+5}}$

b) $\lim_{n\to\infty}\left(1+\frac{-5}{n-1}\right)^{n+5}=\lim_{n\to\infty}\left[\left(1+\frac{-5}{n-1}\right)^{n+5}\right]^{\frac{n-1}{n+5}}$

c) $\lim_{n\to\infty}\left(1+\frac{-5}{n-1}\right)^{n+5}=\lim_{n\to\infty}\left[\left(1+\frac{-5}{n-1}\right)^{n-1}\right]^{\frac{n+5}{n+5}}$

d) $\lim_{n\to\infty}\left(1+\frac{-5}{n-1}\right)^{n+5}=\lim_{n\to\infty}\left[\left(1+\frac{-5}{n-1}\right)^{n-1}\right]^{\frac{n+5}{n-1}}$

Question 7

$$\lim_{n\to\infty}\left[\left(1+\frac{-2}{n^2-n+1}\right)^{n^2-n+1}\right]^{\frac{3n^2+5}{n^2-n+1}}=\boxed{?}$$

$$\lim_{n\to\infty}\frac{3n^2+5}{n^2-n+1}=3$$

What is the value of the above limit?

a) e^{-3}

b) e^{3}

c) e^{-6}

d) e^{-2}

Question 8

$$\lim_{n\to\infty}\left(\frac{3n+5-5+2}{3n+5}\right)^{7n}$$

How should we split the expression into two fractions so that we can apply the formula for *e*?

a) $\lim_{n\to\infty}\left(\frac{3n+5}{3n+5}-\frac{5+2}{3n+5}\right)^{7n}$

b) $\lim_{n\to\infty}\left(\frac{3n+5}{3n+5}-\frac{-5-2}{3n+5}\right)^{7n}$

c) $\lim_{n\to\infty}\left(\frac{3n+5}{3n+5}+\frac{-5+2}{3n+5}\right)^{7n}$

d) $\lim_{n\to\infty}\left(\frac{3n+5}{3n+5}+\frac{5-2}{3n+5}\right)^{7n}$

Question 9

$$\lim_{n\to\infty}\left(1+\frac{4n-1}{n^2+5}\right)^{n^2+5}$$

At this point can we already use the *e*-formula to state the result?

a) Yes
b) No, the expression still needs to be transformed

Question 10

$$\lim_{n\to\infty}\left(2+\frac{2}{n}\right)^{n}=$$

What is the value of the above limit?

a) e^2
b) e
c) 0
d) ∞

Part 2: EXERCISES

Ex. 1

Solve the following limits:

1) $\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{n}$

2) $\lim_{n\to\infty}\left(1+\frac{4}{n^2}\right)^{n^2}$

3) $\lim_{n\to\infty}\left(1-\frac{7}{n}\right)^{n}$

4) $\lim_{n\to\infty}\left(1-\frac{1}{n-2}\right)^{n-2}$

5) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+100}$

6) $\lim_{n\to\infty}\left(1-\frac{5}{n^2+3n}\right)^{2n^2+7}$

7) $\lim_{n\to\infty}\left(\frac{n+5}{n+1}\right)^{n-4}$

8) $\lim_{n\to\infty}\left(\frac{5n}{5n-2}\right)^{7n+11}$

9) $\lim_{n\to\infty}\left(\frac{2n^2-1}{2n^2+2}\right)^{6n^2-1}$

10) $\lim_{n\to\infty}\left(\frac{1-n^2}{4-n^2}\right)^{7n}$

END