LIMITS COURSE

LESSON 4

Three-Sequence Theorem.

Sums of arithmetic and geometric sequences.

HOMEWORK

Part 1: TEST

Select the correct answer (only one is true).

Question 1

$$\lim_{n\to\infty}\sqrt[n]{3^4+5^4+7^4}$$

How can this limit be calculated?

a) Use the Three-Sequence Theorem.
b) Split the expression into three separate roots; the result will be 3+5+7=15.
c) Rewrite the powers under the root so they all have the same base.
d) Use the fact that $\lim_{n\to\infty}\sqrt[n]{a}=1$. The result will be 1.

Question 2

What does it mean that the sequence a_n bounds the sequence b_n from below?

a) Every term of b_n is greater than every term of a_n
b) The sequence a_n tends to a smaller limit than b_n
c) Every term of a_n is greater than or equal to the corresponding term of b_n
d) Every term of b_n is greater than or equal to the corresponding term of a_n

Question 3

$$\lim_{n\to\infty}\sqrt[n]{4^n+5^n}$$

To evaluate this limit we want to use the Three-Sequence Theorem. Which sequence should be chosen as the lower bound?

a) $\sqrt[n]{4^n}$
b) $\sqrt[n]{4^n+4^n}$
c) $\sqrt[n]{5^n}$
d) $\sqrt[n]{5^n+5^n}$

Question 4

$$\lim_{n\to\infty}\sqrt[n]{3^n+3^n+3^n+3^n}$$

To what value does this sequence converge?

a) to 4
b) to ∞
c) to 0
d) to 3

Question 5

$$S_n = a_1\frac{1-q^n}{1-q}$$

The above formula is the formula for …

a) a geometric sequence
b) the sum of a geometric sequence
c) the limit of a geometric sequence
d) the product of a geometric sequence

Question 6

$$\lim_{n\to\infty}\frac{2+4+8+16+\ldots 2^n}{10+16+22+\ldots+(6n-2)}$$

Which formulas should be used to find the limit of the above sequence?

a) Use the geometric-series sum formula in the numerator and the arithmetic-series sum formula in the denominator.
b) Use the arithmetic-series sum in the numerator and the geometric-series sum in the denominator.
c) Use only the arithmetic-series sum formula.
d) Use only the geometric-series sum formula.

Question 7

$$\lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{4}+\dots\right)$$

Can the limit of the above sequence be found with the geometric-series sum formula?

a) No
b) Yes

Question 8

$$\lim_{n\to\infty}\left(1+1\tfrac{1}{2}+2+2\tfrac{1}{2}+\dots\right)$$

Can the limit of the above sequence be found with the arithmetic-series sum formula?

a) No
b) Yes

Question 9

$$\lim_{n\to\infty}\frac{n^2\cos n}{n^3+n^2-1}$$

We want to apply the Three-Sequence Theorem to this limit. Which sequence should we use to bound it from above?

a) $\lim_{n\to\infty}\frac{n^2\cos 1}{n^3+n^2-1}$
b) $\lim_{n\to\infty}\frac{n^2(-1)}{n^3+n^2-1}$
c) $\lim_{n\to\infty}\frac{n^3\cdot 1}{n^3+n^2-1}$
d) $\lim_{n\to\infty}\frac{n^2}{n^3+n^2-1}$

Question 10

$$\frac{1}{n^2+10}$$

Which of the following expressions is smaller than the above term of the sequence ?

a) $\frac{n}{n^2+10}$

b) $\frac{2}{n^2+10}$

c) $\frac{1}{n^2+11}$

d) $\frac{1}{n^2+9}$

Part 2: EXERCISES

Ex. 1

Solve the following limits:

1) $\lim_{n\to\infty}\sqrt[n]{2^n+4^n+8^n}$

2) $\lim_{n\to\infty}\sqrt[n]{\left(\frac{5}{6}\right)^n+\left(\frac{3}{4}\right)^n+\left(\frac{11}{12}\right)^n}$

3) $\lim_{n\to\infty}\sqrt[n]{3^n+7^n+11^n+5^n}$

4) $\lim_{n\to\infty}\frac{4}{n^2}\left(1+2+\ldots+n\right)$

5) $\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{3}{n^2}+\frac{5}{n^2}+\ldots+\frac{2n-1}{n^2}\right)$

6) $\lim_{n\to\infty}\frac{1+\frac{1}{4}+\frac{1}{16}+\ldots+\frac{1}{4^{n-1}}}{4}$

7) $\lim_{n\to\infty}\left(\frac{1+2+\ldots+n}{n+2}-\frac{n}{2}\right)$

8) $\lim_{n\to\infty}\frac{\sin n!}{n}$

9) $\lim_{n\to\infty}\frac{n^2+\sin(n-1)}{2n^2+2}$

10) $\lim_{n\to\infty}\left[n\left(\frac{1}{n^2+1}+\frac{1}{n^2+2}+\frac{1}{n^2+3}+\ldots+\frac{1}{n^2+n}\right)\right]$

END