LIMITS
COURSE

LESSON 7
Limit formulas for functions

HOMEWORK

Part 1: TEST

Select the correct answer (only one is true).

Question 1

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{x} = 1$$

Is the above limit computed correctly?

a) Yes
b) No

Question 2

$$\lim_{x\to 0} x\cot x =$$

How should this limit be evaluated correctly?

a) $\lim_{x\to 0} x\cot x = \lim_{x\to 0} x\frac{\cot x}{x} x = 0$

b) $\lim_{x\to 0} x\cot x = \lim_{x\to 0} x\frac{\cos x}{\sin x} = \lim_{x\to 0} x\frac{\cos x}{\frac{\sin x}{x} x} = 1$

c) $\lim_{x\to 0} x\cot x = \lim_{x\to 0} x\frac{\tan x}{x} x = 0$

d) $\lim_{x\to 0} x\cot x = \lim_{x\to 0} x\frac{\sin x}{\cos x} = \lim_{x\to 0} x\frac{\frac{\sin x}{x} x}{\cos x} = 0$

Question 3

$$\lim_{x\to 0}\frac{2x}{\sin 4x}$$

How should this limit be evaluated correctly?

a) $\lim_{x\to 0}\frac{2x}{\sin 4x}=\lim_{x\to 0}\frac{2x}{\frac{\sin 4x}{4x}}4x=0$

b) $\lim_{x\to 0}\frac{2x}{\sin 4x}=\lim_{x\to 0}\frac{2x}{\frac{\sin 4x}{2x}2x}=1$

c) $\lim_{x\to 0}\frac{2x}{\sin 4x}=\lim_{x\to 0}\frac{2x}{\frac{\sin 4x}{4x}4x}=\frac{1}{2}$

d) $\lim_{x\to 0}\frac{2x}{\sin 4x}=\lim_{x\to 0}\frac{2x}{\frac{\sin 4x}{4}4}=\frac{1}{2}$

Question 4

$$\lim_{x\to 0}\frac{\sin 3x-\sin 2x}{\sin x}=\lim_{x\to 0}\frac{\frac{\sin 3x}{3x}3x-\frac{\sin 2x}{2x}2x}{\frac{\sin x}{x}x}=\lim_{x\to 0}\frac{1\cdot 3x-1\cdot 2x}{1\cdot x}=\lim_{x\to 0}\frac{x}{x}=1$$

Why is the above limit computation incorrect?

a) Because it gives the wrong result
b) Because incorrect limit formulas were used
c) Because it contains algebraic/arithmetical errors
d) Because $x\to 0$ was not applied consistently in all steps (only in the 2nd and 3rd transitions)

Question 5

$$\lim_{x\to 0}\frac{\arcsin x}{\arctan x}$$

While evaluating this limit we will use the fact that…

a) $\lim_{x\to 0}\frac{\sin x}{x}=1$

b) $\lim_{x\to 0}\frac{\arcsin x}{x}=1$

c) $\lim_{x\to 0}\frac{\operatorname{arccot} x}{x}=1$

d) $\lim_{x\to 0}\frac{\tan x}{x}=1$

Question 6

$$\lim_{x\to 0}\frac{e^{-x}-1}{x}=1$$

Is the above result correct?

a) Yes

b) No

Question 7

$$\lim_{x\to 0}\frac{10^{2x}-1}{2x}=\boxed{?}$$

What is the value of this limit?

a) 1

b) 2

c) $\ln 10$

d) $\ln 2$

Question 8

$$\frac{\sin\square}{\square}$$

What conditions are sufficient for the whole expression to converge to 1 ?

a) The two boxed expressions must be equal and their variable *x* must tend to 0
b) The two boxed expressions must be equal
c) The two boxed expressions must be equal and must themselves tend to 0
d) The two boxed expressions must tend to 0

Question 9

It is true that …

a) $1-\cos x=\frac{1}{1-\cos x}$
b) $1-\cos x=\frac{1}{\frac{1}{1-\cos x}}$
c) $1-\cos x=1-\frac{1}{\cos x}$
d) $1-\cos x=\sin x$

Question 10

$$\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$$

To what does this limit converge ?

a) ∞
b) The limit does not exist
c) 1
d) 0

Part 2: EXERCISES

Ex. 1

Solve the following limits:

1) $\lim\limits_{x\to 0}\dfrac{\sin 2x}{x}$
2) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{8x}$
3) $\lim\limits_{x\to 0}\dfrac{4x}{\sin 5x}$
4) $\lim\limits_{x\to 0}\dfrac{\sin 10x}{\sin 5x}$
5) $\lim\limits_{x\to 0}\dfrac{\tan 4x}{x}$
6) $\lim\limits_{x\to 0}\dfrac{\arctan 4x}{3x}$
7) $\lim\limits_{x\to 0}\dfrac{\arcsin 5x}{\sin 5x}$
8) $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x}$
9) $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{x}$
10) $\lim\limits_{x\to 0}\dfrac{3^{4x^2}-1}{7x^2}$
11) $\lim\limits_{x\to 0}\dfrac{\ln(1+5x)}{x}$
12) $\lim\limits_{x\to 0}\dfrac{\log_2(1+x)}{2x}$
13) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{e^x-1}$
14) $\lim\limits_{x\to 0}\dfrac{4^{-2x}-1}{2x}$
15) $\lim\limits_{x\to 0^+}\dfrac{\ln\left(1+\sqrt{x}\right)}{\sqrt[3]{x}}$
16) $\lim\limits_{x\to 0}\sin 3x\cot 5x$

17) $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{\ln(1+4x)}$

18) $\lim\limits_{x\to \pi}\dfrac{\sin 7x}{\sin 5x}$

19) $\lim\limits_{x\to 0}\dfrac{\sin 7x-\sin 5x}{\sin x}$

20) $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{\sin^3 x}$

21) $\lim\limits_{x\to 0}\dfrac{\sin 2x-\sin 4x}{\sin x-\sin 2x}$

22) $\lim\limits_{x\to 3}\dfrac{\arctan(3x-9)}{x^2-9}$

END