LIMITS COURSE

LESSON 8
One-sided limits of functions.
Continuity of functions.

HOMEWORK

Part 1: TEST

Select the correct answer (only one is true).

Question 1

$$\lim_{x\to 1^-} f(x) = -2$$
$$\lim_{x\to 1^+} f(x) = -2$$

What follows from the two equalities above?

a) That the limit of the function at x=1 does **not** exist
b) That $f(x) = -2$
c) That $\lim_{x\to 1} f(x) = -2$
d) That $\lim_{x\to 1} f(x) = 1$

Question 2

$$\lim_{x\to 0^-} f(x)$$

What does the above notation tell us??

a) That the function takes negative values
b) That the function values approach zero from the left side
c) That the limit of the function tends to zero
d) That the arguments x are negative

Question 3

$$\lim_{x\to 1} \frac{1}{(x-1)^2}$$

How should this limit be evaluated correctly?

a) Compute one-sided limits of the function
b) Substitute x=1 and read the result from the form $\left[\frac{A}{0}\right] = \pm\infty$
c) It is impossible
d) Apply the factoring method

Question 4

$$\lim_{x\to 2^-}\frac{|2-x|}{2-x}$$

How should this limit be evaluated??

a) $\lim_{x\to 2^-}\frac{|2-x|}{2-x}=\lim_{x\to 2^-}\frac{2-x}{2-x}=1$
b) $\lim_{x\to 2^-}\frac{|2-x|}{2-x}=\lim_{x\to 2^-}\frac{-(2-x)}{2-x}=-1$
c) $\lim_{x\to 2^-}\frac{|2-x|}{2-x}=\lim_{x\to 2^-}\frac{x-2}{2-x}=\lim_{x\to 2^-}\frac{x-2}{-(x-2)}-1$
d) This limit does not exist

Question 5

$$\lim_{x\to 0^-}\left(\frac{1}{2}\right)^x$$

To what does this limit tend?

a) $+\infty$
b) $-\infty$
c) 0
d) 1

Question 6

$$f(x)=\begin{cases}4x-1 & \text{for } x>1\\ 5x-2 & \text{for } x\le 1\end{cases}$$

What is $f(0)=\ldots?$

a) -1
b) -2
c) 1
d) 0

Question 7

To guarantee that a function is continuous at a point x_0 it is sufficient that …

a) the left-hand and right-hand limits at that point are equal
b) the left-hand and right-hand limits at that point are defined and the function value exists there
c) the limit of the function exists at that point
d) both one-sided limits at that point are equal to the function value at that point

Question 8

What does it mean that a function is *continuous*?

a) It is continuous at all of its points of discontinuity
b) It can be drawn with a pen without lifting it
c) It is continuous at every point of its domain
d) The phrase has no meaning

Question 9

$$f(x)=\begin{cases} 2 & \text{for } x\in(-\infty,-1) \\ x^2 & \text{for } x\in\langle -1,1\rangle \\ e^x & \text{for } x\in(1,\infty) \end{cases}$$

At which points must we test continuity to decide whether the function is continuous?

a) 0 and 1
b) -1 and 1
c) 2 and e
d) 0 and -1

Question 10

Can a function be both continuous and discontinuous at the same time?

a) Yes
b) No

Part 2: EXERCISES

Ex. 1

Solve the following limits:

1) $\lim\limits_{x \to -1^-} \frac{2x}{x+1}$
2) $\lim\limits_{x \to -2^-} \frac{4x+2}{x+2}$
3) $\lim\limits_{x \to -4^+} \frac{x}{|x+4|}$
4) $\lim\limits_{x \to -4^-} \frac{x}{|x+4|}$
5) $\lim\limits_{x \to 2^-} \frac{2x}{2-x}$
6) $\lim\limits_{x \to 5^+} \frac{2x+5}{25-x^2}$
7) $\lim\limits_{x \to 0^+} 3^{\frac{1}{x}}$
8) $\lim\limits_{x \to 2^+} \left(e^x + \frac{1}{x-2} \right)$
9) $\lim\limits_{x \to 3^-} \frac{3x}{e^{\frac{x}{3-x}} + 2}$
10) $\lim\limits_{x \to 4} \frac{1}{(x-4)^2}$

Ex. 2

Examine the continuity of the function:

1) $f(x) = \begin{cases} -2 & \text{for } x \le 0 \\ \frac{x^3 - 2x}{x^2 + 1} & \text{for } x > 0 \end{cases}$
2) $f(x) = \begin{cases} x+1 & \text{for } x \le 1 \\ \frac{x-5}{x-1} & \text{for } x > 1 \end{cases}$

3) $f(x)=\begin{cases}\frac{5-x}{25-x^2} & \text{for } x<5\\ \frac{1}{2x} & \text{for } x\geq 5\end{cases}$

4) $f(x)=\begin{cases}4x+\frac{1}{6} & \text{for } x<0\\ \frac{1}{6} & \text{for } x=0\\ \frac{\sqrt{x+9}-3}{x} & \text{for } x>0\end{cases}$

5) $f(x)=\begin{cases}\frac{x+2}{4-x^2} & \text{for } x<2\\ x^2 & \text{for } x=2\\ 4^{\frac{x}{x-2}} & \text{for } x>2\end{cases}$

6) $f(x)=\begin{cases}x^2 & \text{for } x\geq 2\\ 2x & \text{for } -2<x<2\\ \frac{x^2}{2} & \text{for } x\leq -2\end{cases}$

Ex. 3

Determine the values of the parameters for which the function is continuous

1) $f(x)=\begin{cases}2x-a & \text{for } x\leq 2\\ 2x^2+1 & \text{for } x>2\end{cases}$

2) $f(x)=\begin{cases}2x+1 & \text{for } x<1\\ x^2-ax & \text{for } x\geq 1\end{cases}$

3) $f(x)=\begin{cases}x & \text{for } x<-1 \text{ and for } x>1\\ x^2+ax+b & \text{for } -1\leq x\leq 1\end{cases}$

END