COURSE ON MATRICES

LESSON 8
System of linear equations
(Gaussian elimination)

HOMEWORK

Part 1: TEST

Select the correct answer (only one is true).

Question 1

$$\begin{cases} x+2y-z=2 \\ x+2y-2z=-1 \\ 3x+2y+z=0 \end{cases}$$
$a)$

$$\begin{cases} x-y=0 \\ 2x+2y=1 \end{cases}$$
$b)$

$$\begin{cases} 3x_1-x_2-2x_3+2x_4=7 \\ x_1+4x_2+3x_3-6x_4=3,5 \end{cases}$$
$c)$

$$\begin{cases} x+y=1 \\ x+2y=4 \\ 2x+y=2 \\ x-y=0 \\ 2x+2y=4 \end{cases}$$
$d)$

Which of the following systems of linear equations cannot be solved using Gaussian elimination?

a) system d)
b) system c)
c) systems c) i d)
d) Any of the above systems can be solved using Gaussian elimination, because it is a universal method

Question 2

$$\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix}$$

$$\begin{bmatrix} -1 & 2 & 3 & 0 & -1 \\ 0 & 3 & 2 & 0 & 0 \\ 0 & 0 & -4 & 2 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

In the row echelon matrix below (with variables x_1, x_2, x_3, x_4, x_5), the variables above the "staircase edges" are:

a) x_1, x_2, x_3, x_5
b) x_1, x_2, x_3, x_4, x_5
c) x_1, x_2, x_3, x_4
d) There are no such variables

Question 3

Can Cramer's systems be solved using Gaussian elimination?

a) Yes, in every case
b) Yes, but only if certain assumptions are satisfied
c) No

Question 4

If during the elementary operations in Gaussian elimination, in some row we get only zeros…

a) The system has no solutions — we stop and write the answer
b) The system definitely has infinitely many solutions
c) We create a non-zero element in that row and continue (until obtaining row echelon form)
d) We cross out the row and continue (until obtaining row echelon form)

Question 5

If during the elementary operations in Gaussian elimination, in some row we get only zeros, except for the last column (the constants) "after the bar", where the element is non-zero…

a) The system has no solutions — we stop and write the answer
b) The system definitely has infinitely many solutions
c) We create a non-zero element in that row and continue
d) We cross out the row and continue

Question 6

An indeterminate system is a system…

a) …, that has no solutions.
b) …, whose solution requires introducing parameters.
c) …, that cannot be solved.
d) …, that can be solved using Cramer's rule.

Question 7

$$\begin{array}{cccc|c} x & y & z & t & \\ 1 & 3 & 0 & 11 & -2 \\ 0 & -2 & 14 & 5 & -2 \\ 0 & 0 & 1 & 0 & 1 \end{array}$$

After applying elementary operations, we obtained the row echelon form shown. Which variable should be replaced by a parameter at this point?

a) None
b) z and t.
c) t
d) y, z and t

Question 8

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ 1 & 3 & 4 & 0 & 5 \\ 0 & 1 & 3 & -2 & -2 \end{array}$$

After applying elementary operations, we obtained the **row echelon** matrix shown. We set $x_3 = \alpha_1$ and $x_4 = \alpha_2$. What will be the first equation read from this matrix?

a) $0x_1 + 3x_2 + 3x_3 - 0x_4 = 5$
b) $0x_1 + 1x_2 + 3\alpha_1 - 2\alpha_2 = -2$
c) $0x_1 + 1x_2 + 3x_3 - 2x_4 = -2$
d) $0x_1 + 1x_2 + 3\alpha_1 + 2\alpha_2 = -2$

Question 9

What does “checking” in Gaussian elimination consist of?

a) Substituting the computed values into one equation of the system
b) Substituting the computed values into every equation of the system
c) Checking whether the values are integers
d) Replacing parameters with zeros in the result

Question 10

$$\begin{cases} x_1 + x_2 = 0 \\ x_1 + x_2 + x_3 = 1 \\ x_2 + x_3 + x_4 = 0 \\ x_3 + x_4 + x_5 = 0 \\ x_4 + x_5 = 1 \end{cases}$$

What will the matrix for this system look like?

$$a)\quad \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \end{matrix}\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \\ 0 & 2 & 3 & 4 & 0 & 0 \\ 0 & 0 & 3 & 4 & 5 & 0 \\ 0 & 0 & 0 & 4 & 5 & 1 \end{array}\right]$$

$$c)\quad \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \end{matrix}\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

$$b)\quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right]$$

$$d)\quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]$$

Part 2: EXERCISES

Ex. 1

Solve systems of equations:

1) $\begin{cases} 2x+y=3 \\ x-y=-6 \end{cases}$

2) $\begin{cases} x-y=3 \\ 2x-2y=7 \end{cases}$

3) $\begin{cases} 2x-y+z=2 \\ 3x+2y+2z=-2 \\ x-2y+z=1 \end{cases}$

4) $\begin{cases} x+y+z+t=4 \\ -x+y+z+t=2 \\ -x-y+z+t=0 \\ -x-y-z+t=-2 \end{cases}$

5) $\begin{cases} 3x-y+z=2 \\ 6x-2y+2z=1 \end{cases}$

6) $\begin{cases} x+y+2z=4 \\ x+y-z=1 \\ 2x+2y+z=5 \end{cases}$

7) $\begin{cases} 3x-2y-5z=3 \\ x+4y-11z=1 \end{cases}$

8) $\begin{cases} x+y+z=2 \\ y+z+2t+v=1 \end{cases}$

9) $\begin{cases} 2x+y+z=1 \\ 3x-y+3z=2 \\ x+y+z=0 \\ x-y+z=1 \end{cases}$

10) $\begin{cases} 2x_1-3x_2+5x_3+7x_4=1 \\ 4x_1-6x_2+2x_3+3x_4=2 \\ 2x_1-3x_2-11x_3-15x_4=1 \end{cases}$

11) $\begin{cases} x-2y+z+t-u=0 \\ 2x+y-z-t+u=0 \\ x+7y-5z-5t+5u=0 \\ 3x-y-2z+t-u=0 \end{cases}$

12) $\begin{cases} x_1 - 2x_2 + 3x_3 - 4x_4 + 2x_5 = -2 \\ x_1 + 2x_2 - x_3 - x_5 = -3 \\ x_1 - x_2 + 2x_3 - 3x_4 = 10 \\ x_2 - x_3 + x_4 - 2x_5 = -5 \\ 2x_1 + 3x_2 - x_3 + x_4 + 4x_5 = 1 \end{cases}$

13) $\begin{cases} 3x + 2y + 2z + 2t = 2 \\ 2x + 3y + 2z + 5t = 3 \\ 9x + y + 4z - 5t = 1 \\ 2x + 2y + 3z + 4t = 5 \\ 7x + y + 6z - t = 7 \end{cases}$

14) $\begin{cases} 2x_4 + 5x_5 = 12 \\ x_3 - 16x_4 + 2x_5 = -11 \\ -x_2 - 13x_3 - 2x_4 + x_5 = -14 \\ 3x_1 + 7x_2 - x_3 - 3x_4 + 2x_5 = 10 \\ x_1 + 2x_2 - 5x_3 + 4x_4 + x_5 = 4 \end{cases}$

END